

Oefening 12 (hyperbolische schroeflijn)

$$x = a \cosh t$$

$$y = a \sinh t \quad a = \text{constante}$$

$$z = at$$

$$\text{Ruimtelijke kromme} \Rightarrow R = (x''^2 + y''^2 + z''^2)^{-1/2}$$

$$\frac{ds}{dt} = v = |\dot{\mathbf{r}}| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

$$= \sqrt{a^2 \sinh^2 t + a^2 \cosh^2 t + a^2}$$

$$= a \sqrt{\sinh^2 t + \cosh^2 t + 1}$$

$$\Downarrow \quad \cosh^2 t - \sinh^2 t = 1$$

$$\frac{ds}{dt} = \sqrt{2}a \cosh t$$

$$x' = \frac{dx}{ds} = \frac{dx}{dt} \frac{dt}{ds} = \frac{dx}{dt} \frac{1}{\frac{ds}{dt}}$$

$$\begin{aligned}
&= a \sinh t \cdot \frac{1}{\sqrt{2}a \cosh t} = \frac{1}{\sqrt{2}} \frac{\sinh t}{\cosh t} \\
y' &= \frac{a \cosh t}{\sqrt{2}a \cosh t} = \frac{1}{\sqrt{2}} \\
z' &= \frac{a}{\sqrt{2}a \cosh t} = \frac{1}{\sqrt{2}} \frac{1}{\cosh t}
\end{aligned}$$

$$\begin{aligned}
x'' &= \frac{d}{ds} \left(\frac{dx}{ds} \right) = \frac{dx'}{dt} \frac{1}{\frac{ds}{dt}} \\
&= \frac{1}{\sqrt{2} \cosh^2 t} \frac{1}{\sqrt{2}a \cosh t} = \frac{1}{2a \cosh^3 t} \\
y'' &= 0 \\
z'' &= -\frac{\sinh t}{2a \cosh^3 t}
\end{aligned}$$

$$\begin{aligned}
x''^2 + y''^2 + z''^2 &= \frac{1}{4a^2 \cosh^6 t} + \frac{\sinh^2 t}{4a^2 \cosh^6 t} \\
&= \frac{\cosh^2 t}{4a^2 \cosh^6 t} = \frac{1}{4a^2 \cosh^4 t}
\end{aligned}$$

\Downarrow

$$R = 2a \cosh^2 t$$

 \Downarrow

$$k = \frac{1}{R} = \frac{1}{2a \cosh^2 t}$$